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COMPRESSIBLE BOUNDARY-LAYER FLOW AT A THREE-DIMENSIONAL STAGNATION POINT WITH MASSIVE BLOWING

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Abstract—The effect of massive blowing rates on the steady laminar compressible boundary-layer flow with variable gas properties at a 3-dim. stagnation point (which includes both nodal and saddle points of attachment) has been studied. The equations governing the flow have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique for nodal points of attachment but employing a parametric differentiation technique instead of quasilinearization for saddle points of attachment. It is found that the effect of massive blowing rates is to move the viscous layer away from the surface. The effect of the variation of the density–viscosity product across the boundary layer is found to be negligible for massive blowing rates but significant for moderate blowing rates. The velocity profiles in the transverse direction for saddle points of attachment in the presence of massive blowing show both the reverse flow as well as velocity overshoot.

NOMENCLATURE

a, b,	velocity gradients in the x and y directions, respectively;
c,	ratio of velocity gradients, b/a;
$C_f, \bar{C}_f$,	skin-friction coefficients in the x and y directions, respectively;
f,	dimensionless stream function such that $f' = u/u_e$;
f_w,	mass transfer parameter, $-(\rho w)_w/(\rho_e\mu_e a)^{1/2}$;
f''_w,	skin-friction parameter in the x direction;
g,	dimensionless enthalpy, h/h_e;
g_w,	dimensionless enthalpy at the wall (wall temperature), h_w/h_e;
h,	enthalpy;
H,	total enthalpy;
N,	ratio of the density–viscosity product across the boundary layer, $\rho\mu/\rho_e\mu_e = g^{\omega-1}$;
Pr,	Prandtl number;
q_w,	local heat-transfer rate at the wall;
Re_x,	local Reynolds number, ax^2/ν_e;
St,	Stanton number (heat-transfer parameter);
T,	temperature;
u, v, w,	velocity components in the x, y, z directions, respectively;
x, y, z,	principal, transverse and normal directions, respectively.

Greek symbols

η,	similarity variable, $(\rho_e a/\mu_e)^{1/2}\int_0^z (\rho/\rho_e)\,dz$;
μ,	coefficient of viscosity;
ν_e,	kinematic viscosity at the edge of the boundary layer;
ρ,	density;
τ_x, τ_y,	shear stresses at the wall in the x and y directions, respectively;
φ,	dimensionless stream function such that $\varphi' = v/v_e$;
φ''_w,	skin-friction parameter in the y direction;
ω,	exponent in the power-law variation of viscosity.

Superscripts

$'$,	prime denotes differentiation with respect to η.

Subscripts

e, w,	denote conditions at the edge of the boundary layer and on the surface $\eta = 0$, respectively;
∞,	free stream value.

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INTRODUCTION

THE PROBLEM of massive blowing (injection) into compressible 3-dim. boundary layers is of considerable interest in the analysis of thermal protection systems of vehicles for the Jovian probe [1]. In actual situations, 3-dim. boundary-layer flows originate at a stagnation point where the convective heating is maximum. It is known that the heating rates can be reduced considerably by injecting large amounts of fluid. For large blowing rates, the structure of the boundary layer is considerably different from that of moderate or no blowing rate. In this case, the boundary layer consists of a relatively thick inner layer having constant shear and temperature and a thin free-fixing type of outer layer, which adjusts the former to match the external inviscid flow. Since this corresponds to zero heat transfer to the surface and greatly reduced surface shear, the results may be considered as the boundary layer having been blown off the surface. In such a situation, the usual methods for treating two-point boundary-value problems are either poorly convergent or nonconvergent. This failure is due to the diminution of shear and heat transfer near the wall and to the increasing extent of the boundary layer normal to the wall. In order to overcome this difficulty, several methods both approximate and exact (numerical) have been developed. These methods include the matched asymptotic expansion method, the quasilinearization method, the matrix method, the backward shooting method, and the implicit finite-difference scheme in combination with the quasilinearization technique.

Kubota and Fernandez [2], Kassoy [3], Vimala and Nath [4], and Libby [5] have used the method of matched asymptotic expansion to study the effect of massive blowing on the steady laminar compressible 2-dim., axisymmetric and 3-dim. stagnation point boundary-layer flows with constant gas properties ($\rho \propto T^{-1}, \mu \propto T$). Libby [5] has also studied the effect of moderate injection rates ($f_w \geq -3$) on the steady 3-dim. stagnation-point flows with constant gas properties. His analysis was subsequently extended by Nath and Meena [6] to include variable gas properties. Both have used the quasilinearization technique for the solution of the governing equations.

Recently, two numerical methods for the solution of boundary-layer equations have been developed by Keller and Cebeci [7] and Wu and Libby [8]. Both methods result in a block tridiagonal matrix. Subsequently, Liu and Davy [9] have found that these methods give accurate results only for low blowing rates ($f_w > -4$). Nachtsheim and Green [10] and Liu and Nachtsheim [11] have used the backward shooting method to study the effect of large blowing rates on steady laminar compressible stagnation-point boundary-layer flows. The backward shooting method essentially reduces to solving a three-point boundary-value problem starting from the dividing streamline. Although this method is stable, its computing time is long and increases as blowing rates increase. In order to overcome the difficulties of the backward shooting method, Liu and Chiu [12] have recently developed an implicit finite-difference scheme in combination with a quasilinearization technique to study the effect of large blowing rates. This method is found to be fast as well as stable and the rate of convergence (and therefore computing time) is independent of blowing rates. Thus, amongst all the methods, the foregoing method seems to be most suited to boundary-layer problems with massive blowing rates. Recently, this method was applied to study the combined effect of large blowing rates and magnetic field with variable gas properties on an axisymmetric stagnation point [13].

Libby and Cresci [14] have shown experimentally that even for large rates of blowing the inviscid flow in the stagnation region is not affected and, therefore, the boundary layer concept can be applied to study the effect of large blowing rates at the stagnation-point flow field without introducing any appreciable error in the analysis.

The aim of the present analysis is to study the effect of massive blowing rates ($-60 \leq f_w < 0$) on steady laminar compressible 3-dim. stagnation-point boundary-layer flow with variable gas properties ($\rho \propto T^{-1}, \mu \propto T^{\omega}$) both for nodal and saddle points of attachment. It may be remarked that the foregoing problem for saddle-point flows ($-1 \leq c < 0$) has been studied only for small blowing rates ($f_w \geq -1.25$) [15]. Even for nodal-point flows ($0 \leq c \leq 1$), the results are available only for moderate blowing rates ($f_w \geq -3$) [5, 6]. Here, the equations governing the flow have been solved numerically using the finite-difference method in combination with the quasilinearization technique [12] for nodal points of attachment ($0 \leq c \leq 1$), but using the parametric differentiation technique [16–18] instead of the quasilinearization technique for saddle points of attachment ($-1 \leq c < 0$). The results for moderate or no blowing rates ($-3 \leq f_w \leq 0$) have been compared with those of Libby [5], Nath and Meena [6], Wortman *et al.* [19], and Nath and Muthanna [20] and for massive blowing rates ($-60 \leq f_w < -3$) with those of Krishnaswamy and Nath [13].

The boundary layer analysis is not strictly applicable to the high-energy viscous-shock layer type of flow field requiring massive blowing for controlling the heat-load to the surface, because there is no asymptotic approach to the velocity profile to some edge value due to the occurrance of strong shock [i.e. $u \to u_s$ (velocity behind the shock) and not $u \to u_e$ (velocity at the edge of the boundary layer)]. However, there is a shear-layer edge within the viscous-shock layer which for the perfect gas case may be defined as $H \to H_s$. In spite of this shortcoming, the present analysis is expected to exhibit most of the characteristics of the massive blowing rates on the flow field and can form the basis of an analysis based on a more realistic model.

GOVERNING EQUATIONS

The equations governing the steady laminar compressible boundary-layer flow of a gas with variable properties (i.e. $\rho \propto T^{-1}$, $\mu \propto T^{\omega}$) in the neighbourhood of the stagnation-point of a 3-dim. body taking into account the effect of blowing can be expressed in dimensionless form as [5, 6, 15]

$$(Nf'')' + (f + c\varphi)f'' + g - f'^2 = 0, \quad (1a)$$

$$(N\varphi'')' + (f + c\varphi)\varphi'' + c(g - \varphi'^2) = 0, \quad (1b)$$

$$Pr^{-1}(Ng')' + (f + c\varphi)g' = 0. \quad (1c)$$

The appropriate boundary conditions are

$$f(0) = f_w (f_w < 0), f'(0) = 0, f'(\infty) \to 1, \quad (2a)$$

$$\varphi(0) = \varphi'(0) = 0,\ \varphi'(\infty) \to 1, \tag{2b}$$

$$g(0) = g_w,\ g(\infty) \to 1. \tag{2c}$$

The parameter c represents the nature of the 3-dim. stagnation points. For nodal points of attachment, $c \geq 0$ $(0 \leq c \leq 1)$ and for saddle points of attachment, $c < 0$ $(-1 \leq c < 0)$. Also $c = 0$ and 1 for 2-dim. and axisymmetric stagnation-point flows, respectively. It may be noted that $\omega = 0.5$ for high temperature flows, $\omega = 0.7$ for low-temperature flows and $\omega = 1.0$ represents the simplification of a constant density–viscosity product [19]. We have taken the Prandtl number Pr to be constant, since its variation in boundary layer, for most atmospheric flight problems, is quite small [19].

The skin-friction coefficients in the x and y directions are given by [6]

$$C_f = 2\tau_x/\rho_e u_e^2 = 2(Re)_x^{-1/2} g_w^{\omega-1} f''_w, \tag{3a}$$

$$\bar{C}_f = 2\tau_y/\rho_e u_e^2 = 2(Re_x)^{-1/2}(v_e/u_e) g_w^{\omega-1} \varphi''_w. \tag{3b}$$

Similarly, the heat-transfer coefficient in terms of Stanton number can be expressed as [6]

$$St = q_w/[(h_e - h_w)\rho_e u_e] = (Re_x)^{-1/2} Pr^{-1} (1 - g_w)^{-1} g_w^{\omega-1} g'_w. \tag{3c}$$

RESULTS AND DISCUSSION

The set of equations (1) has been solved numerically under conditions (2) using an implicit finite-difference scheme in combination with the quasilinearization technique. Since the method is described in full detail by Liu and Chiu [12], its description is omitted here. This method gives results for large rates of blowing $(-60 \leq f_w < 0)$ for nodal points of attachment $(0 \leq c \leq 1)$. However, for saddle points $(-1 \leq c < 0)$, this method does not converge for large rates of blowing. In order to overcome this difficulty, we have used the method of parametric differentiation in combination with the finite-difference scheme for saddle points $(-1 \leq c < 0)$. Using the results for $c = 0$ (obtained by the implicit finite-difference scheme in combination with quasilinearization) as the starting values for the parameter c, we have obtained the solution of equations (1) under conditions (2) for various values of c in the range $-1 \leq c < 0$ by the method of parametric differentiation. The resulting equations (which are linear) have been solved numerically using an implicit finite-difference scheme. Since the method of parametric differentiation is also described in great detail elsewhere [16–18], it is not presented here.

Computations have been carried out for various values of the parameters. Variable step size has been used in the η direction starting with a large step size and reducing it uniformly as it moves towards the dividing streamline. For moderate blowing rates $(f_w \geq -3)$, the starting step size $h_1 = 0.2$ and the last step size $h_N = 0.05$ have been used and further reduction in these step sizes does not alter the results up to four decimal places. For large blowing rates $(f_w < -3)$, the starting step size $h_1 = 0.8$ and the last step size $h_N = 0.1$ have been used and further reduction in them has no effect on the results up to the fifth decimal place. For parametric differentiation with respect to the parameter c, a constant step size $\Delta c = -0.05$ has been used and further reduction in the step size has no effect on the results up to the fifth decimal place. Moreover, the value of η_∞ (edge of the boundary layer) is chosen depending upon the rate of blowing. η_∞ increases as the blowing rate increases. For example when $f_w = -30$, we have taken $\eta_\infty = 70$ and when $f_w = -60$, $\eta_\infty = 100$ has been used. Further change in η_∞ does not affect the results up to the fifth decimal place.

In order to test the accuracy of the method, we have compared our results for moderate rates of blowing $(-3 \leq f_w < 0)$ with those obtained by the quasilinearization technique [5, 6], the operator technique [19] and by parametric differentiation [20] and they are found to be in excellent agreement. However, for the sake of brevity, comparison only with those of Libby [5] and Nath and Meena [6] has been given in Tables 1 and 2. In Fig. 1, we have also compared the velocity profiles (f', φ') for $c = 0.25$ and $f_w = -3$ (moderate blowing rate) with those obtained by the method of matched asymptotic expansion [5] and we find significant difference in the results in a certain range of η (the maximum difference (in φ') is about 30% at $\eta \simeq 12.4$). However, this difference can be reduced by taking more terms in the matched asymptotic expansion. Also, the velocity and enthalpy profiles (f', φ', g) for massive blowing rates $(-60 \leq f_w < -3)$ for $c = 1$ have been compared with those of Krishnaswamy and Nath [13] and they are found to be in excellent agreement (Fig. 2).

The effect of massive blowing $(f_w \leq -10)$ on the velocity and enthalpy profiles (f', φ', g) for nodal-point flows $(0 \leq c \leq 1)$ has been shown in Figs. 2 and 3 and for saddle-point flows $(-1 \leq c < 0)$ in Figs. 4–6. For saddle-point flows, the velocity profiles in the x direction (f') show overshoot whereas the velocity profiles in the y direction (φ') show reverse flow and overshoot (Figs. 4 and 5). This behaviour is due to the combined effects of inertia, pressure and shear stress. The magnitude of the overshoot in f' and φ' as well as the region of reverse flow in φ' increase as the blowing rate f_w increases. On the other hand, for nodal-point flows $(0 \leq c \leq 1)$, there is neither reverse flow nor velocity overshoot in either of the profiles (Figs. 2 and 3). For zero or moderate blowing rates $(f_w \geq -3)$, the velocity profiles in the y direction (φ') exhibit reverse flow but no overshoot in the saddle-point region $(-1 \leq c < 0)$. Similar behaviour has been observed by Libby [15]. Here, the profiles for $f_w \geq -3$ are not shown for the sake of brevity*. It is observed that for both nodal and saddle point flows $(-1 \leq c \leq 1)$, there is a rapid increase in the thickness of the boundary layer but a rapid decrease in wall shear and heat

* They may be obtained from the authors.

Table 1. A comparison of skin-friction and heat-transfer parameters for $g_w = 0.1$, $\omega = 1$ and $Pr = 0.7$

	$c = 1$			$c = 0$		
$-f_w$	f''_w	φ''_w	$g'_w/(1 - g_w)$	f''_w	φ''_w	$g'_w/(1 - g_w)$
0	0.8460 (0.8481)*	0.8460 (0.8481)	0.6128 (0.6122)	0.6761 (0.6750)	0.5058 (0.5050)	0.4310 (0.4230)
1	0.2764 (0.2767)	0.2764 (0.2767)	0.1840 (0.1844)	0.1648 (0.1650)	0.0420 (0.0420)	0.0671 (0.0670)
2	0.0531 (0.0531)	0.0531 (0.0531)	0.0043 (0.0043)	0.0499 (0.050)	0.0001 (0.0)	0.0 (0.0)
3	0.0333 (0.0333)	0.0333 (0.0333)	0.0 (0.0)	0.0333	0.0	0.0

*The values in the parentheses are the values obtained by Libby [5].

Table 2. A comparison of skin-friction and heat-transfer parameters for $c = 0.5$ and $Pr = 0.7$

		$g_w = 0.2$			$g_w = 0.6$		
$-f_w$	ω	f''_w	φ''_w	$g'_w/(1 - g_w)$	f''_w	φ''_w	$g'_w/(1 - g_w)$
1	0.5	0.2208 (0.2209)*	0.1634 (0.1635)	0.1027 (0.1025)	0.5042 (0.5028)	0.3413 (0.3414)	0.1619 (0.1618)
	1.0	0.2889 (0.2888)	0.2060 (0.2061)	0.1329 (0.1330)	0.5478 (0.5468)	0.3662 (0.3663)	0.1727 (0.1730)
2	0.5	0.1044 0.1043)	0.0591 (0.0590)	0.0161 (0.0160)	0.2928 (0.2930)	0.1600 (0.1601)	0.0250 (0.0250)
	1.0	0.1008 (0.1008)	0.0517 (0.0517)	0.0029 (0.0029)	0.2967 (0.2965)	0.1578 (0.1581)	0.0169 (0.0173)
3	0.5	0.0664 (0.0665)	0.0338 (0.0336)	0.0008 (0.0008)	0.1985 (0.1987)	0.1010 (0.1011)	0.0012 (0.0012)
	1.0	0.0666 (0.0665)	0.0334 (0.0334)	0.0 (0.0)	0.1988 (0.1989)	0.1005 (0.1005)	0.0003 (0.0003)

*The values in the parentheses are the values obtained by Nath and Meena [6].

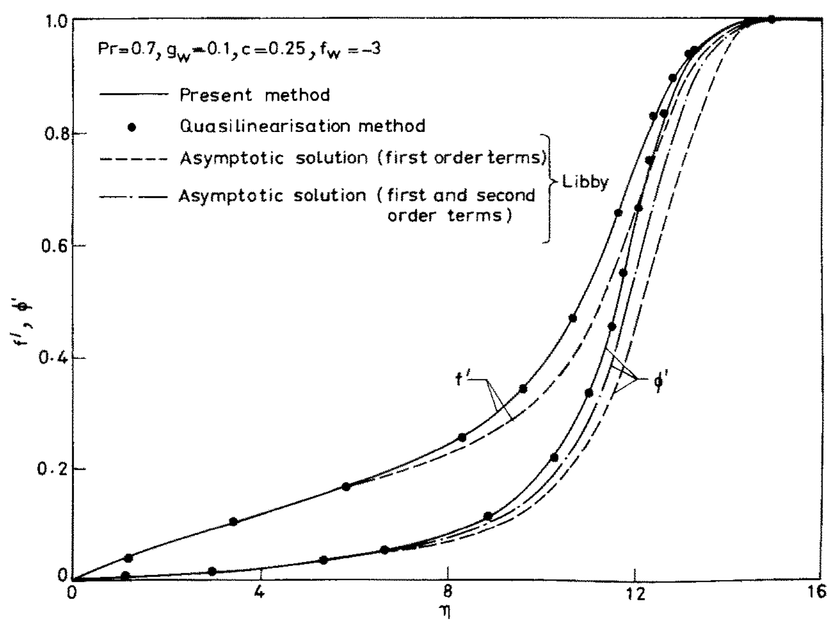

FIG. 1. Comparison of velocity profiles for moderate blowing ($c = 0.25$).

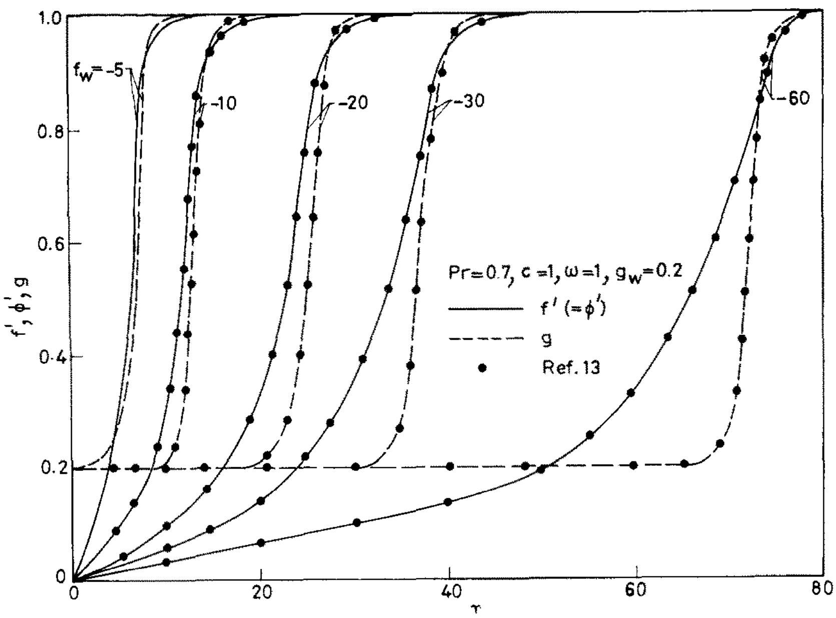

FIG. 2. Comparison of velocity and enthalpy profiles for large blowing (c = 1.0).

transfer with increasing blowing rates. Also there is an enthalpy overshoot in the saddle-point region for massive blowing rates (Fig. 6). The overshoot in the enthalpy profiles is due to the massive blowing which results in a very slight decrease of temperature below the wall temperature in the saddle-point region and hence the heat-transfer parameter $g'_w = 0^-$. However, the zero heat-transfer condition ($g'_w = 0$) corresponds to unit Prandtl number ($Pr = 1$) but here we have taken $Pr = 0.7$. Because of this difference of Prandtl number and a slight decrease of temperature below the wall condition, the enthalpy for the boundary-layer flow exceeds that in the outer inviscid flow for some range. This is analogous to the enthalpy overshoot observed by Yasuhara [21] for $g_w = 1$ and $Pr \neq 1$. There is no such phenomenon in the nodal-point region for massive blowing as there is no decrease of enthalpy below the specified value at the wall and hence the heat-transfer parameter $g'_w = 0^+$ in the nodal-point region.

The variations of the skin-friction and heat-transfer parameters (f''_w, φ''_w, g'_w) with c ($-1 \leq c \leq 1$) for

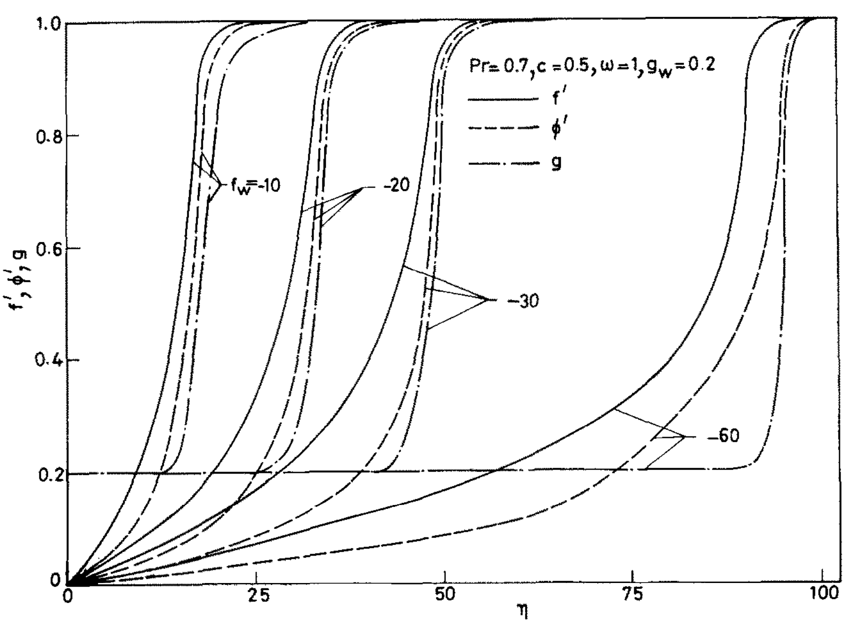

FIG. 3. Effect of large blowing on velocity and enthalpy profiles (c = 0.5).

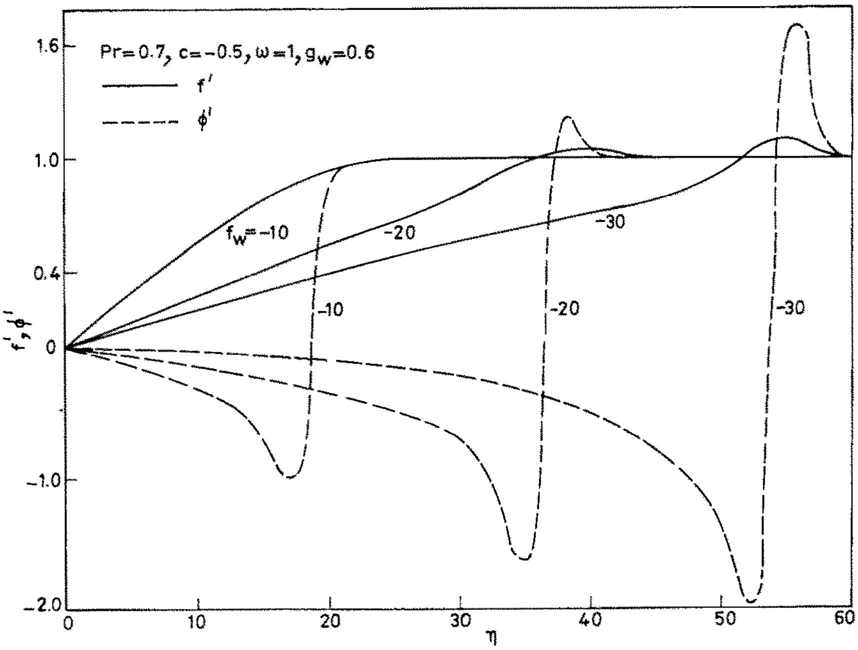

FIG. 4. Effect of large blowing on velocity profiles ($c = -0.5$).

moderate and large blowing rates are shown in Figs. 7 and 8, respectively. For moderate blowing rates (Fig. 7), f''_w and g'_w decrease as c decreases until at some negative c, φ''_w is reversed and f''_w and g'_w begin to increase as c decreases. This trend has also been observed by Libby [15], Wortman *et al.* [19], and Nath and Muthanna [20]. Figure 7 also shows the effect of the variation of the density-viscosity product across the boundary layer characterized by the parameter ω on f''_w, φ''_w, and g'_w. The effect of ω becomes less pronounced as blowing rate increases and for large blowing, the effect is almost negligible (Fig. 8). Also for large blowing, f''_w and g'_w ($g'_w \simeq 0$) become almost insensitive to the change in c, however, φ''_w decreases as c decreases.

The effect of c on dividing streamline velocities $(f')_{f+c\varphi=0}$, $(\varphi')_{f+c\varphi=0}$, and enthalpy $(g)_{f+c\varphi=0}$ is shown in Fig. 9. They are found to be almost insensitive

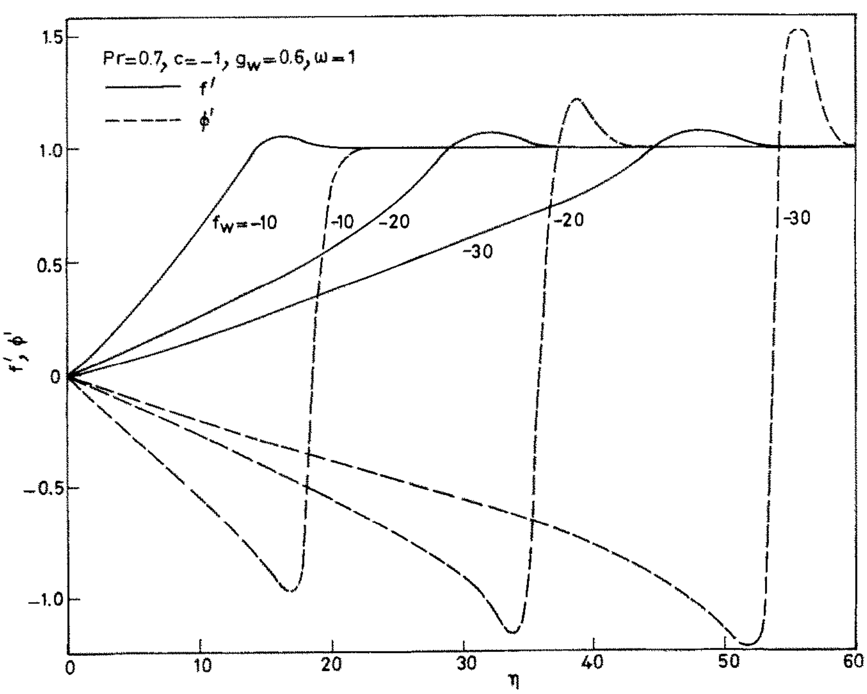

FIG. 5. Effect of large blowing on velocity profiles ($c = -1.0$).

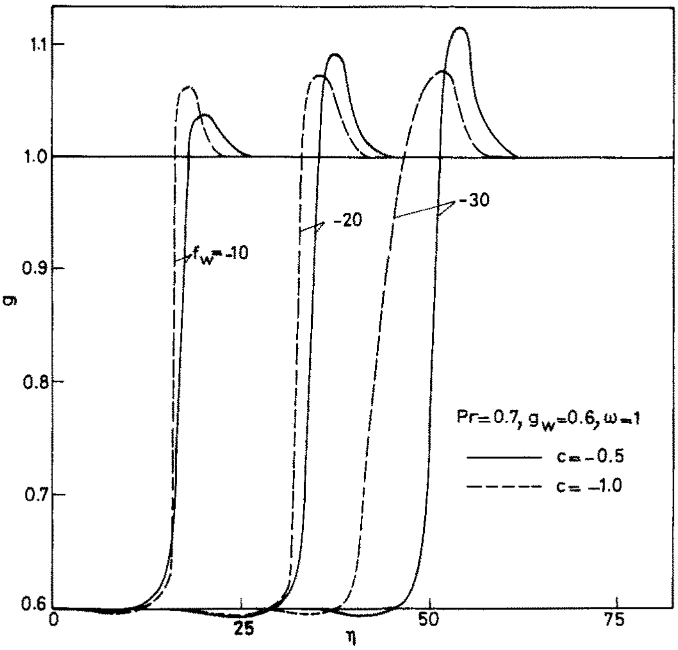

FIG. 6. Effect of large blowing on enthalpy profiles ($c = -0.5, -1.0$).

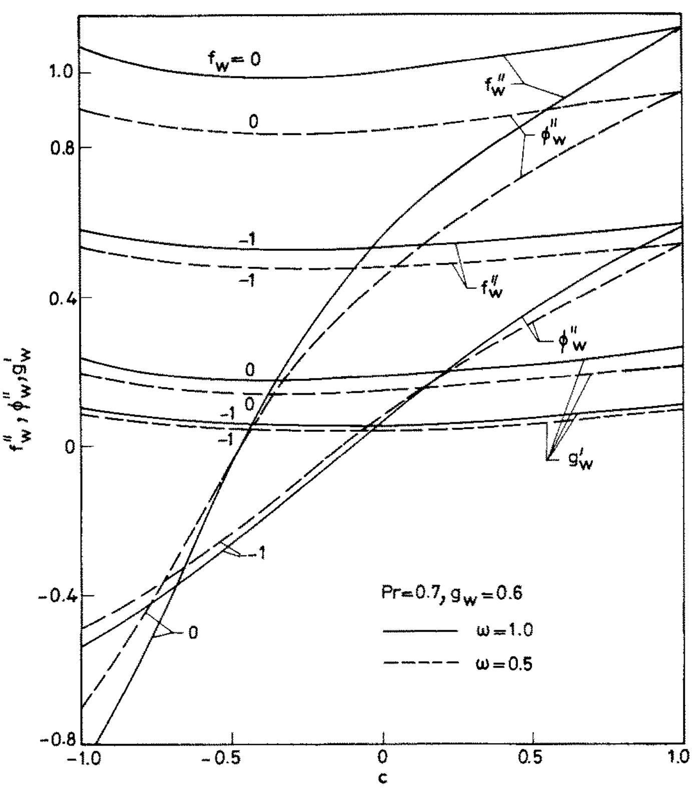

FIG. 7. Effect of ω on skin-friction and heat-transfer parameters ($f_w = 0, -1.0$).

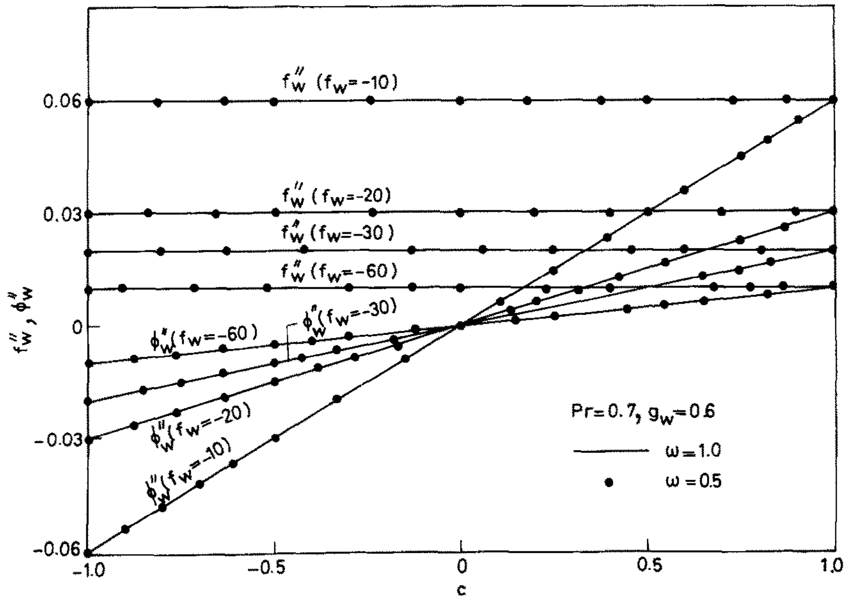

FIG. 8. Effect of ω on skin-friction and heat transfer parameters $(-60 \leq f_w < -1.0)$.

to the change in c in the nodal-point region $(0 \leq c \leq 1)$. However, in the saddle-point region $(-1 \leq c \leq 0)$, the effect of c is more pronounced, especially on $(g)_{f+c\varphi=0}$. As the blowing rate increases, $(f')_{f+c\varphi=0}$, $(\varphi')_{f+c\varphi=0}$, and $(g)_{f+c\varphi=0}$ also increase for a certain range of c, but they decrease beyond this range.

The variation of the location of the dividing streamline $(\eta)_{f+c\varphi=0}$ for the entire range of c $(-1 \leq c \leq 1)$ is shown in Fig. 10. It increases up to a certain maximum level for some value of c and then decreases. It is noted that the increase in the blowing rate shifts the dividing streamline away from the boundary.

The effect of wall temperature g_w on the dividing streamline velocity and enthalpy profiles for large blowing rate is shown in Fig. 11. For nodal-point flows $(c = 0.5)$, the dividing streamline velocity in the x direction and enthalpy profiles increase as g_w increases. For saddle-point flows $(c = -0.5)$, they decrease as g_w increases until $g_w = g_w^*$ and then increase with g_w. The dividing streamline velocity in the y direction $(\varphi')_{f+c\varphi=0}$ increases when the wall temperature g_w is increased slightly from $g_w = 0.1$. But, for further increase in g_w, it decreases rapidly up to a certain value and then increases slowly as g_w tends to 1.

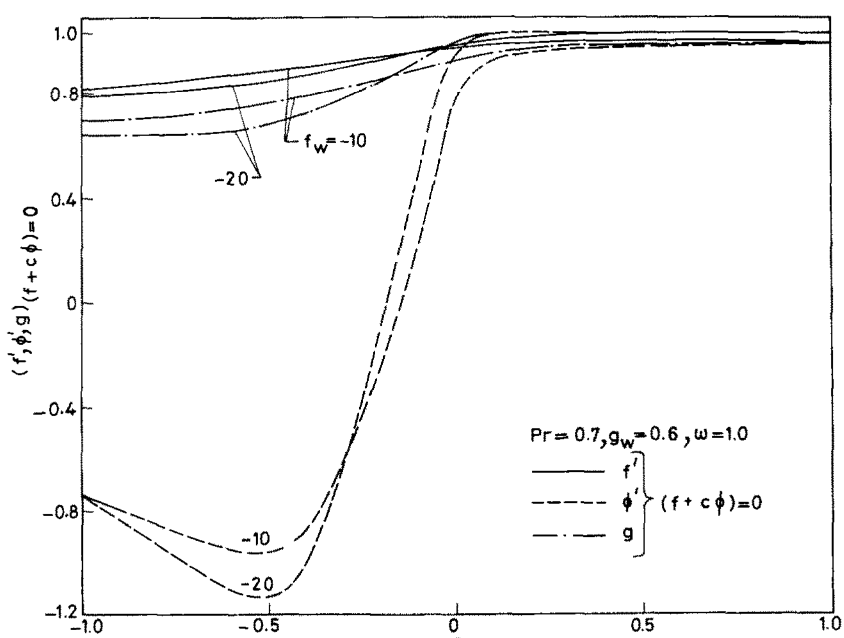

FIG. 9. Effect of the body configuration c on the velocity and enthalpy at the dividing streamline.

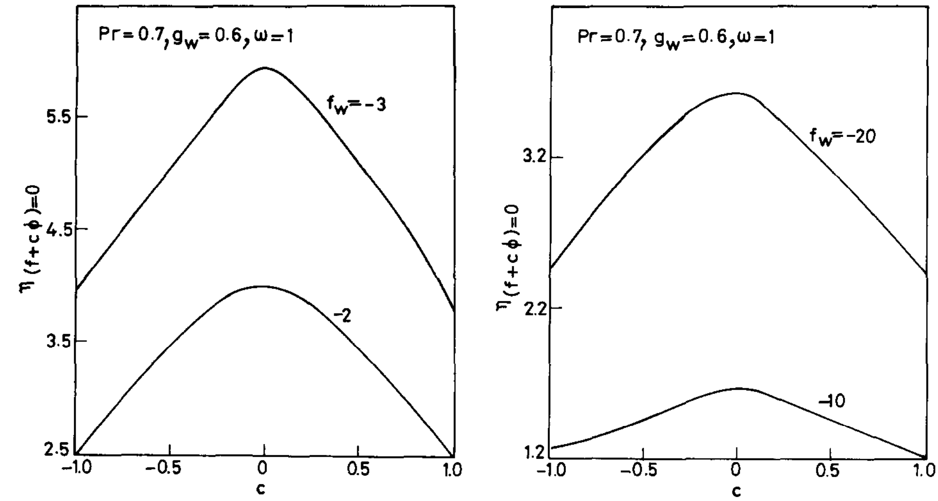

FIG. 10. Variation of the location of the dividing streamline with body configuration c.

The effect of g_w on the location of dividing streamline $(\eta)_{f+c\varphi=0}$ is shown in Fig. 12. It increases as g_w increases. Further, an increase in blowing rate shifts the location of dividing streamline away from the wall for all wall temperature conditions.

CONCLUSIONS

The effect of massive blowing is to move the dividing streamline away from the surface. The effect of the variation of the density–viscosity product across the boundary layer (i.e. variable gas properties) is found to be negligible for large blowing rates. However, they are found to be significant for moderate blowing rates. The velocity and enthalpy profiles in the saddle-point region ($-1 \leq c < 0$) for large blowing rates show some interesting features not encountered in nodal-point region ($0 \leq c \leq 1$). In this region, the velocity profiles in the x direction have velocity overshoot and the velocity profiles in the y direction have both reverse flow and velocity overshoot and they increase as the blowing rates increase. There is also an enthalpy overshoot which increases as the blowing rate increases. For massive blowing case, we find that the boundary layer consists of a relatively thick inner layer

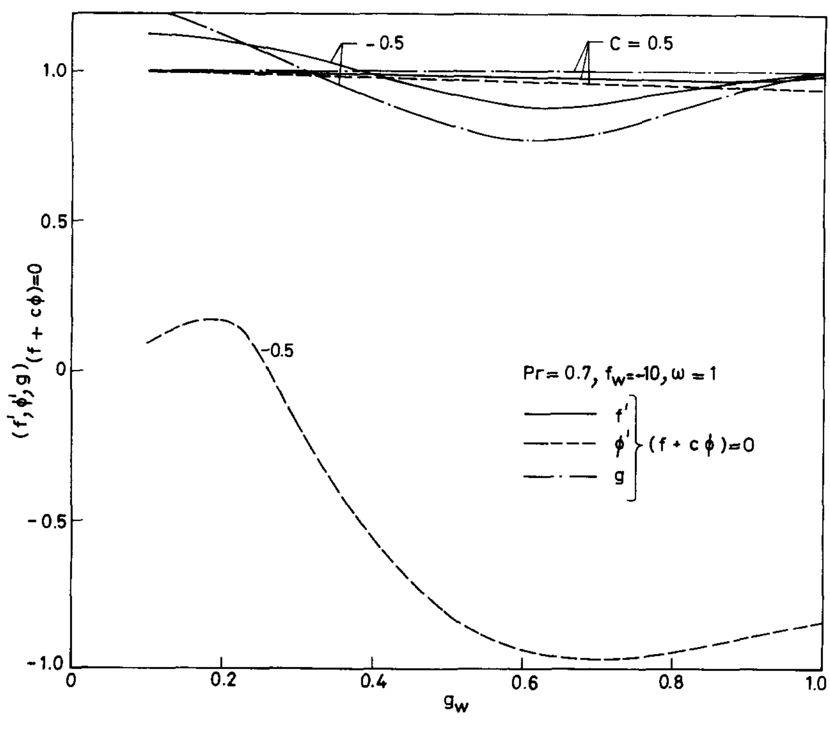

FIG. 11. Effect of wall temperature on the velocity and enthalpy at the dividing streamline.

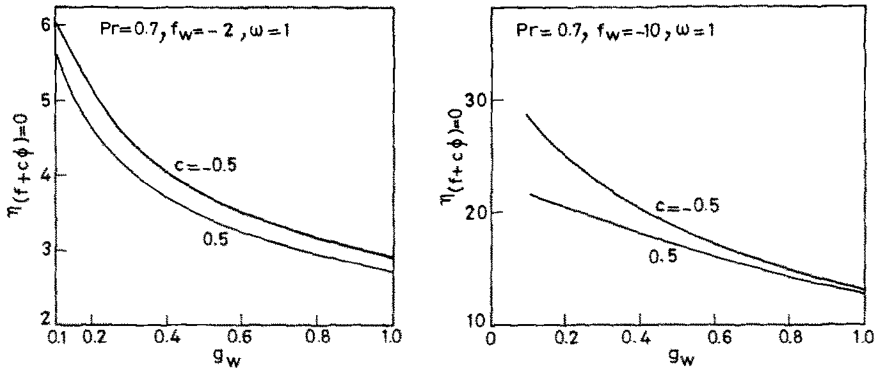

FIG. 12. Variation of the location of the dividing streamline with wall temperature.

with zero heat transfer and negligible skin friction and a relatively thin outer layer adjusting the inner and external flows. Here the results of ref. [12] have been extended to the case of saddle points of attachment by using parametric differentiation in place of quasi-linearization employed in ref. [12]. The present method enables us to obtain the solution exactly for massive blowing rates for both saddle and nodal point regions which could not be obtained by previous investigators.

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ECOULEMENT DE COUCHE LIMITE COMPRESSIBLE AU POINT D'ARRET TRIDIMENSIONNEL ET AVEC SOUFFLAGE

Résumé—On étudie l'effet d'un soufflage important sur l'écoulement de couche limite compressible, avec des propriétés variables du gaz, au point d'arrêt tridimensionnel. Les équations du mouvement sont résolues numériquement en utilisant un schéma implicite aux différences finies en combinaison avec la technique de quasi-linéarisation pour les points nodaux d'attache, mais en employant une technique de différenciation paramétrique à la place de la quasi-linéarisation pour les points relatifs à une selle. On montre que l'effet des grands débits de soufflage est de déplacer la couche limite en l'écartant de la surface. L'effet de la variation du produit masse volumique-viscosité à travers la couche limite est trouvé négligeable pour les grands débits de soufflage, mais il est sensible pour les débits modérés. Les profils de vitesse dans la direction transversale pour les points relatifs à une selle en présence de grands débits de soufflage montrent à la fois un écoulement de retour et une survitesse.

DIE KOMPRESSIBLE GRENZSCHICHTSTRÖMUNG AM DREIDIMENSIONALEN STAUPUNKT BEI STARKEM AUSBLASEN

Zusammenfassung—Der Einfluß starken Ausblasens auf die stationäre laminare kompressible Grenzschicht mit veränderlichen Stoffwerten des Gases am dreidimensionalen Staupunkt (unter Einschluß sowohl konvexer wie konkaver Anlagepunkte) wurde untersucht. Die Strömungsgleichungen wurden numerisch mittels eines implizierten Differenzen-Verfahrens gelöst—für konvexe Anlagepunkte in Verbindung mit dem Quasilinearisierungsverfahren und für konkave Anlagepunkte unter ersatzweiser Verwendung parametrischer Differentiation. Als Auswirkung starken Ausblasens ergibt sich ein Abdrängen der zähen Unterschicht von der Oberfläche. Der Einfluß der Änderung des Dichte-Viskositäts-Produkts quer zur Grenzschicht wird für starkes Ausblasen vernachlässigbar, ist aber bedeutend für mäßiges Ausblasen. Die Geschwindigkeitsprofile in Querrichtung für konkave Anlagepunkte zeigen bei starkem Ausblasen sowohl Strömungsumkehr als auch Geschwindigkeits-Erhöhung.

ТЕЧЕНИЕ СЖИМАЕМОГО ПОГРАНИЧНОГО СЛОЯ В ТРЕХМЕРНОЙ КРИТИЧЕСКОЙ ТОЧКЕ ПРИ ИНТЕНСИВНОМ ВДУВЕ

Аннотация—Исследовано влияние скорости интенсивного вдува на установившееся ламинарное течение сжимаемого газа с переменными свойствами в трехмерной критической точке (включая рассмотрение как узловых, так и седловых точек присоединения потока). Уравнения движения решены численно с использованием неявной конечно-разностной схемы. Использовались также метод квазилинеаризации для узловых точек присоединения и метод параметрической дифференциации (вместо квазилинеаризации) для седловых точек присоединения потока. Найдено, что при интенсивном вдуве вязкий слой оттесняется от поверхности, а изменение произведения плотности на вязкость поперек пограничного слоя практически не влияет в режиме интенсивного вдува, но имеет существенное значение при умеренных скоростях вдува. По профилям скорости для седловых точек присоединения потока при интенсивном вдуве установлена возможность возвратного течения и резкого скачка скорости.